

Put each polynomial in standard form. Then determine the degree, number of terms, end behavior and number of u-turns (humps).

$$1) \quad y = -9x^2(2x + 4x^3 - 9x^2) \\ -18x^3 - 36x^5 + 81x^4$$

$$\text{Stand. Form: } -36x^5 + 81x^4 - 18x^3$$

$$\text{Degree: } 5 \quad \text{Terms: } 3$$

$$\text{End. Beh: } \uparrow\downarrow \quad \text{U-turns: } 4$$

odd \ominus

$$2) \quad y = (3x^2 - 7)(x^2 + 2) \\ 3x^4 + 6x^2 - 7x^2 - 14 \\ 3x^4 - x^2 - 14$$

$$\text{Stand. Form: } 3x^4 - x^2 - 14$$

$$\text{Degree: } 4 \quad \text{Terms: } 3$$

$$\text{End. Beh: } \uparrow\uparrow \quad \text{U-turns: } 3$$

even \oplus

Solve each of the following polynomials.

$$3) \quad x^4 - 8x^3 + 12x^2 = 0 \\ x^2(x^2 - 8x + 12) = 0 \\ x^2(x - 6)(x - 2) = 0 \\ \left. \begin{array}{l} x^2 = 0 \\ x = 0, 0 \end{array} \right\} \left. \begin{array}{l} x - 6 = 0 \\ x = 6 \end{array} \right\} \left. \begin{array}{l} x - 2 = 0 \\ x = 2 \end{array} \right\}$$

$$\text{Roots: } 0, 0, 6, 2$$

$$4) \quad 12x^3 = 10x^2 + 8x$$

$$12x^3 - 10x^2 - 8x = 0$$

$$2x(6x^2 - 5x - 4) = 0$$

$$(6x^2 - 8x)(3x - 4)$$

$$2x(3x - 4) + 1(3x - 4)$$

$$2x(2x + 1)(3x - 4) = 0$$

$$\left. \begin{array}{l} 2x = 0 \\ x = 0 \end{array} \right\} \left. \begin{array}{l} 2x + 1 = 0 \\ x = -\frac{1}{2} \end{array} \right\} \left. \begin{array}{l} 3x - 4 = 0 \\ x = \frac{4}{3} \end{array} \right\}$$

$$\text{Roots: } 0, -\frac{1}{2}, \frac{4}{3}$$

Find the polynomial function, in standard form, given the following roots:

5) $x = 0, 4, -7$

$$\begin{aligned} & x(x-4)(x+7) \\ & x(x^2+3x-28) \\ & x^3+3x^2-28x \end{aligned}$$

Polynomial: $y = x^3 + 3x^2 - 28x$

6) $x = -2, -1, 8$

$$\begin{aligned} & (x+2)(x+1)(x-8) \\ & (x^2+3x+2)(x-8) \\ & x^3+3x^2+2x-8x^2-24x-16 \\ & x^3-5x^2-22x-16 \end{aligned}$$

Polynomial: $y = x^3 - 5x^2 - 22x - 16$

7) $x = 0, 0, 4, -5$

$$\begin{aligned} & x^2(x-4)(x+5) \\ & x^2(x^2+x-20) \\ & x^4+x^3-20x^2 \end{aligned}$$

Polynomial: $y = x^4 + x^3 - 20x^2$

8) $x = 2, -2, 5$

$$\begin{aligned} & (x-2)(x+2)(x-5) \\ & (x^2-4)(x-5) \\ & x^3-5x^2-4x+20 \end{aligned}$$

Polynomial: $y = x^3 - 5x^2 - 4x + 20$

Graph each of the polynomial functions. Be sure to fill in all of the requested information.

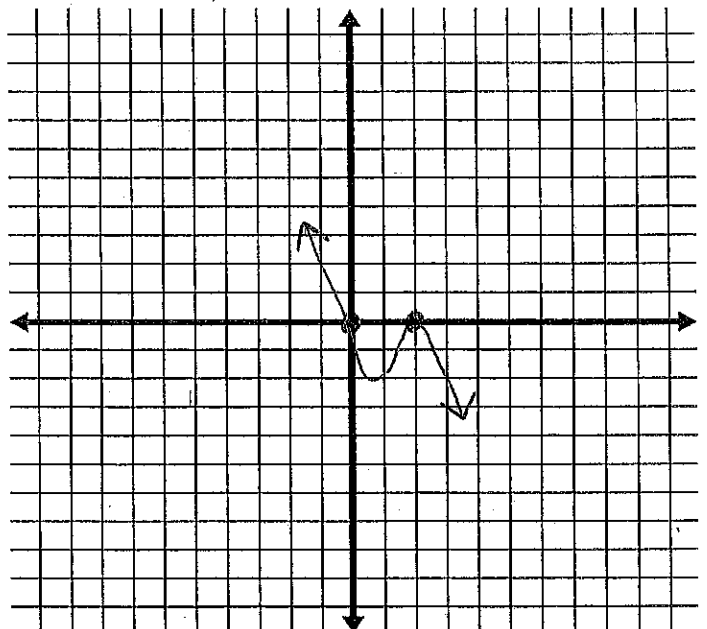
9) $y = -2x^3 + 8x^2 - 8x$

$$\begin{aligned} & -2x(x^2-4x+4) \\ & -2x(x-2)(x-2) \end{aligned}$$

Zeros: 0, 2, 2

y-int: (0, 0)

End Behavior: $\uparrow \downarrow$
odd \ominus



10) $f(x) = x^3 - 7x^2 + 14x - 8$

Maximum # of roots: 3

Possible rational roots: $\pm 1, \pm 2, \pm 4, \pm 8$

Factored form: $(x-1)(x-4)(x-2)$

Actual roots: 1, 4, 2

y-intercept = $(0, -8)$

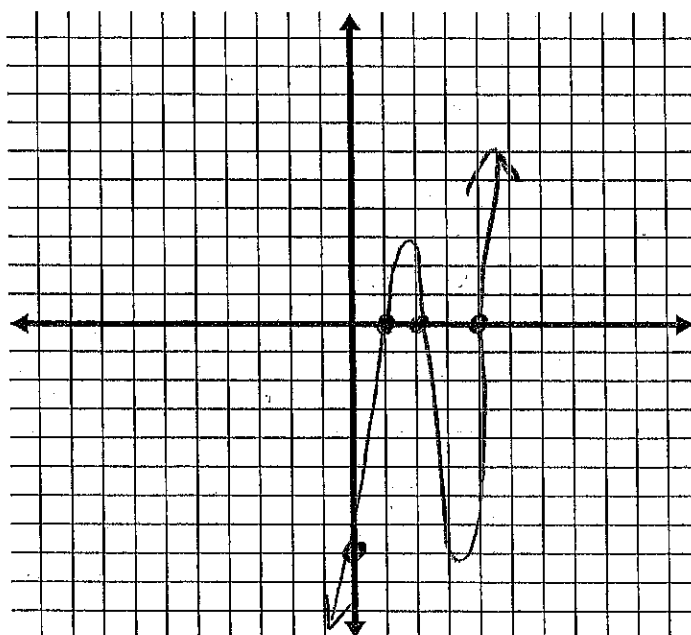
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Workspace:

$$\begin{array}{r} 1 \mid 1 \quad -7 \quad 14 \quad -8 \\ \downarrow \quad 1 \quad -6 \quad 8 \\ \hline 1 \quad -6 \quad 8 \quad 0 \end{array}$$

$$x^2 - 6x + 8$$

$$(x-4)(x-2)$$



11) $f(x) = x^4 - 4x^3 - 13x^2 + 4x + 12$

Maximum # of roots: 4

Possible rational roots: $\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12$

Factored form: $(x-1)(x+1)(x-6)(x+2)$

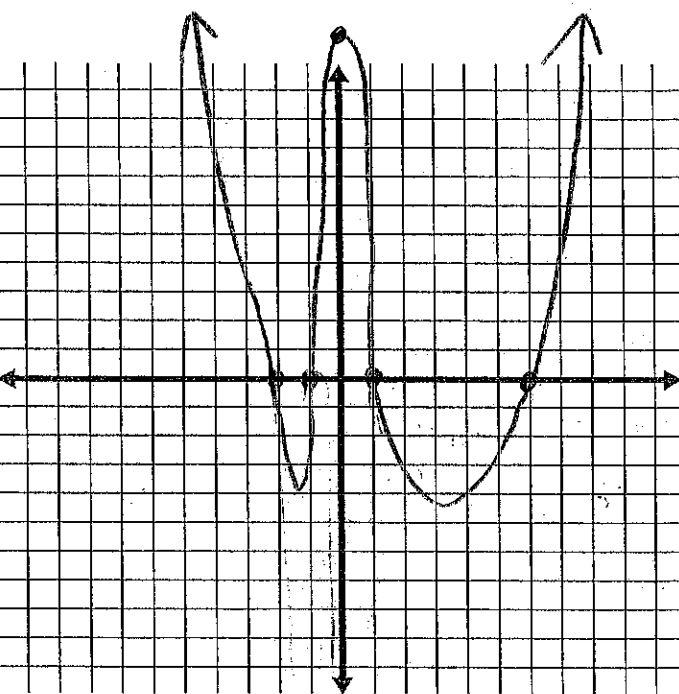
Actual roots: 1, -1, 6, -2

y-intercept = $(0, 12)$

EB: ↑ left, ↑ right.

Workspace:

$$\begin{array}{r} 1 \mid 1 \quad -4 \quad -13 \quad 4 \quad 12 \\ \downarrow \quad 1 \quad -3 \quad -16 \quad -12 \\ \hline -1 \mid 1 \quad -3 \quad -16 \quad -12 \quad 0 \\ \downarrow \quad -1 \quad 4 \quad 12 \\ \hline 1 \quad -4 \quad -12 \quad 0 \end{array}$$



$$x^2 - 4x - 12$$

$$(x-6)(x+2)$$

12) $f(x) = 2x^4 - 11x^3 - 6x^2 + 64x + 32$

Maximum # of roots: 4

Possible rational roots: $\pm 1, \pm 2, \pm 4, \pm 8, \pm 16, \pm 32, \pm \frac{1}{2}$

Factored form: $(x+2)(x-4)(2x+1)(x-4)$

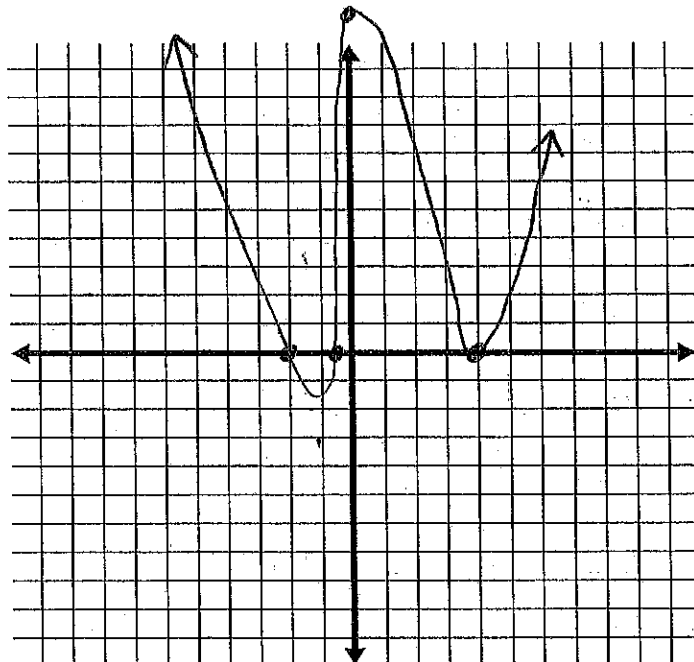
Actual roots: $-2, 4, -\frac{1}{2}, 4$

y-intercept = $(0, 32)$

EB: \uparrow left, \uparrow right.

Workspace:

$$\begin{array}{r} \cancel{2} \mid 2 \quad -11 \quad -6 \quad 64 \quad 32 \\ \quad \downarrow \quad -4 \quad 30 \quad -48 \quad -32 \\ \hline 4 \mid 2 \quad -15 \quad 24 \quad 16 \quad 0 \\ \quad \downarrow \quad 8 \quad -28 \quad -16 \\ \hline 2 \quad -7 \quad -4 \quad 0 \end{array}$$



$$2x^2 - 7x - 4$$

$$(2x+1)(x-4)$$

13) $f(x) = x^3 - 12x + 16$

Maximum # of roots: 3

Possible rational roots: $\pm 1, \pm 2, \pm 4, \pm 8, \pm 16$

Factored form: $(x-2)(x-2)(x+4)$

Actual roots: $2, -4, 2$

y-intercept = $(0, 16)$

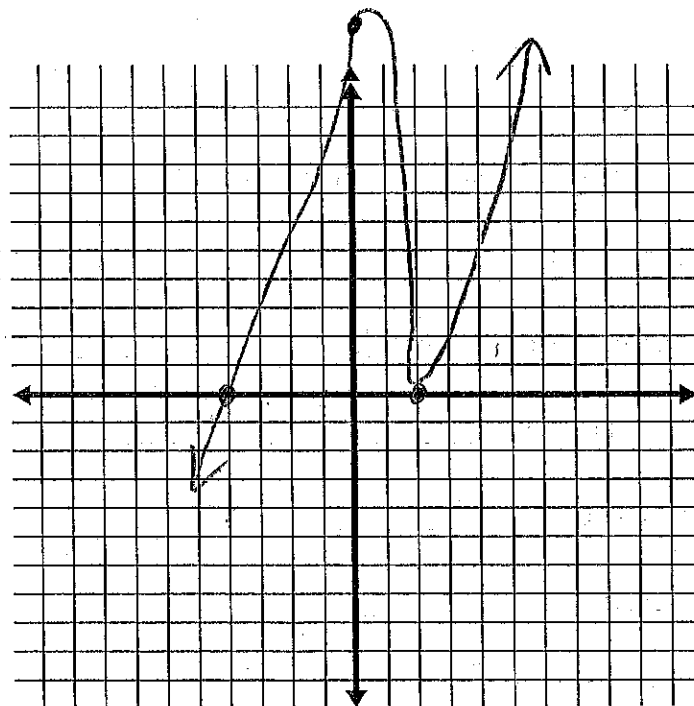
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$$\begin{array}{r} 2 \mid 1 \quad 0 \quad -12 \quad 16 \\ \quad \downarrow \quad 2 \quad 4 \quad -16 \\ \hline 1 \quad 2 \quad -8 \quad 0 \end{array}$$

$$x^2 + 2x - 8$$

$$(x+4)(x-2)$$



Find the requested information:

14) One factor of $f(x) = 4x^3 + 28x^2 - 9x - 63$ is $(x+7)$. What are the other two **factors**?

$$\begin{array}{r} -7 \overline{) 4 \ 28 \ -9 \ -63} \\ \underline{\downarrow -28 \ 0 \ 63} \\ 4 \ 0 \ -9 \ 63 \end{array}$$

$$4x^2 - 9$$

$$(2x+3)(2x-3)$$

Factors: $(2x+3)(2x-3)$

15) The function $f(x) = x^3 - 3x^2 - 28x + 60$ has three x-intercepts. One of the intercepts occurs at $x = -5$. What are the other two **x-intercepts**?

$$\begin{array}{r} -5 \overline{) 1 \ -3 \ -28 \ 60} \\ \underline{\downarrow -5 \ 40 \ -60} \\ 1 \ -8 \ 12 \ 0 \end{array}$$

$$x^2 - 8x + 12$$

$$(x-6)(x-2)$$

$$x-6=0$$

$$x=6$$

$$x-2=0$$

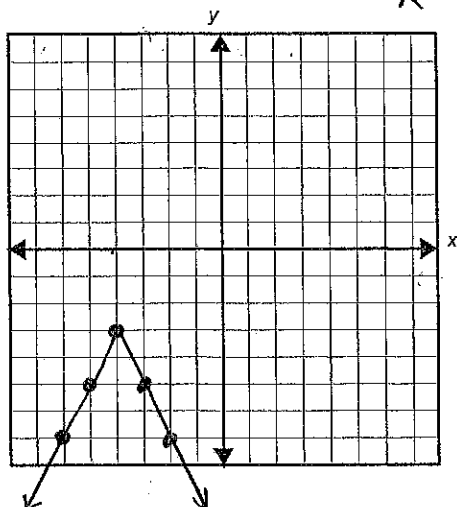
$$x=2$$

x-intercepts: 2, 6

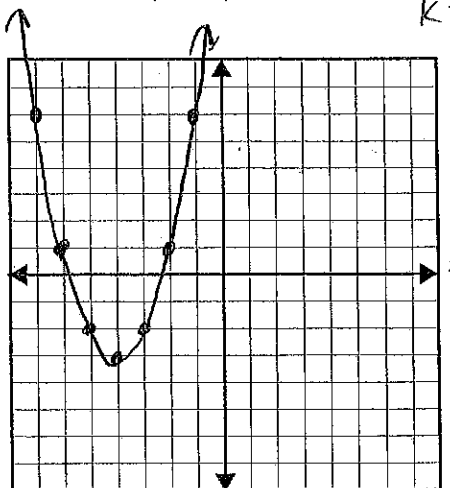
Cumulative Questions:

Graph the following functions using the grids below.

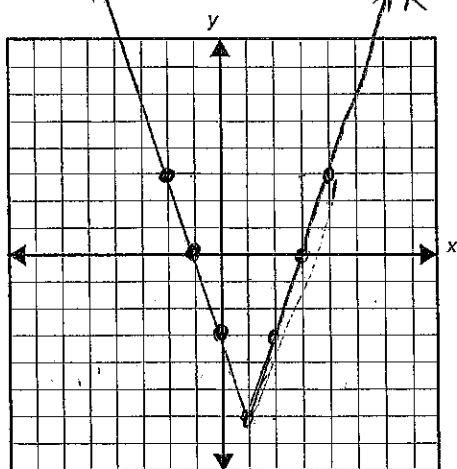
16) $y = -2|x+4| - 3$ $a = -2$
 $h = -4$
 $k = -3$



17) $y = (x+4)^2 - 3$ $a = 1$
 $h = -4$
 $k = -3$



18) $y = 3|x-1| - 6$ $a = 3$
 $h = 1$
 $k = -6$



19) $y = -\frac{1}{2}(x+6)^2 + 4$ $a = -\frac{1}{2}$
 $h = -6$
 $k = 4$

